**Chapter 2: Materials**

**2-1** From Tables A-20, A-21, A-22, and A-24c,
(a) UNS G10200 HR: \( S_u = 380 \) (55) MPa (kpsi), \( S_y = 210 \) (30) MPa (kpsi) \( Ans. \)
(b) SAE 1050 CD: \( S_u = 690 \) (100) MPa (kpsi), \( S_y = 580 \) (84) MPa (kpsi) \( Ans. \)
(c) AISI 1141 Q&T at 540°C (1000°F): \( S_u = 896 \) (130) MPa (kpsi), \( S_y = 765 \) (111) MPa (kpsi) \( Ans. \)
(d) 2024-T4: \( S_u = 446 \) (64.8) MPa (kpsi), \( S_y = 296 \) (43.0) MPa (kpsi) \( Ans. \)
(e) Ti-6Al-4V annealed: \( S_u = 900 \) (130) MPa (kpsi), \( S_y = 830 \) (120) MPa (kpsi) \( Ans. \)

**2-2** (a) Maximize yield strength: Q&T at 425°C (800°F) \( Ans. \)

(b) Maximize elongation: Q&T at 650°C (1200°F) \( Ans. \)

**2-3**
Conversion of kN/m\(^3\) to kg/m\(^3\) multiply by \( 1 \times 10^3 / 9.81 = 102 \)

AISI 1018 CD steel: Tables A-20 and A-5
\[
\frac{S_y}{\rho} = \frac{370}{76.5} = 47.4 \text{ kN} \cdot \text{m/kg} \quad \text{Ans.}
\]

\[
\frac{S_y}{\rho} = \frac{169 \times 10^3}{26.6} = 62.3 \text{ kN} \cdot \text{m/kg} \quad \text{Ans.}
\]

Ti-6Al-4V titanium: Tables A-24c and A-5
\[
\frac{S_y}{\rho} = \frac{830}{43.4} = 187 \text{ kN} \cdot \text{m/kg} \quad \text{Ans.}
\]

ASTM No. 40 cast iron: Tables A-24a and A-5. Does not have a yield strength. Using the ultimate strength in tension
\[
\frac{S_u}{\rho} = \frac{42.5 \times 6.89}{70.6} = 40.7 \text{ kN} \cdot \text{m/kg} \quad \text{Ans}
\]

**2-4**

AISI 1018 CD steel: Table A-5
\[
E = \frac{30.0 \times 10^6}{0.282} = 106 \times 10^6 \quad \text{in Ans.}
\]

2011-T6 aluminum: Table A-5
\[ E = \frac{10.4 \left(10^6\right)}{0.098} = 106 \times 10^6 \text{ in Ans.} \]
Ti-6Al-6V titanium: Table A-5

\[
\frac{E}{\gamma} = \frac{16.5 \times 10^6}{0.160} = 103 \times 10^6 \text{ in} \quad \text{Ans.}
\]

No. 40 cast iron: Table A-5

\[
\frac{E}{\gamma} = \frac{14.5 \times 10^6}{0.260} = 55.8 \times 10^6 \text{ in} \quad \text{Ans.}
\]

2-5

\[
2G (1 + v) = E \quad \Rightarrow \quad v = \frac{E - 2G}{2G}
\]

Using values for \(E\) and \(G\) from Table A-5,

\[
\begin{align*}
\text{Steel:} & \quad v = \frac{30.0 - 2(11.5)}{2(11.5)} = 0.304 \quad \text{Ans.} \\
\text{The percent difference from the value in Table A-5 is} & \quad \frac{0.304 - 0.292}{0.292} = 0.0411 = 4.11 \text{ percent} \quad \text{Ans.}
\end{align*}
\]

\[
\begin{align*}
\text{Aluminum:} & \quad v = \frac{10.4 - 2(3.90)}{2(3.90)} = 0.333 \quad \text{Ans.} \\
\text{The percent difference from the value in Table A-5 is} & \quad 0 \text{ percent} \quad \text{Ans.}
\end{align*}
\]

\[
\begin{align*}
\text{Beryllium copper:} & \quad v = \frac{18.0 - 2(7.0)}{2(7.0)} = 0.286 \quad \text{Ans.} \\
\text{The percent difference from the value in Table A-5 is} & \quad \frac{0.286 - 0.285}{0.285} = 0.00351 = 0.351 \text{ percent} \quad \text{Ans.}
\end{align*}
\]

\[
\begin{align*}
\text{Gray cast iron:} & \quad v = \frac{14.5 - 2(6.0)}{2(6.0)} = 0.208 \quad \text{Ans.} \\
\text{The percent difference from the value in Table A-5 is} & \quad \frac{0.208 - 0.211}{0.211} = -0.0142 = -1.42 \text{ percent} \quad \text{Ans.}
\end{align*}
\]

2-6  \(a\) \(A_0 = \pi (0.503)^2 / 4 = 0.1987 \text{ in}^2, \sigma = P_i / A_0\)
For data in elastic range, \( \dot{l} = \Delta l / l_0 = \Delta l / 2 \)

For data in plastic range \( \dot{l} = \Delta l / \tau = l_l - l_A = \Delta l / A \)

On the next two pages, the data and plots are presented. Figure (a) shows the linear part of the curve from data points 1-7. Figure (b) shows data points 1-12. Figure (c) shows the complete range. \textbf{Note:} The exact value of \( A_0 \) is used without rounding off.

(b) From Fig. (a) the slope of the line from a linear regression is \( E = 30.5 \text{ Mpsi} \) \textit{Ans.}

From Fig. (b) the equation for the dotted offset line is found to be

\[ \sigma = 30.5(10^6) \int - 61000 \]  \hspace{1cm} (1)

The equation for the line between data points 8 and 9 is

\[ \sigma = 7.60(10^6) \int + 42900 \]  \hspace{1cm} (2)

Solving Eqs. (1) and (2) simultaneously yields \( \sigma = 45.6 \text{ kpsi} \) which is the 0.2 percent offset yield strength. Thus, \( S_y = 45.6 \text{ kpsi} \) \textit{Ans.}

The ultimate strength from Figure (c) is \( S_u = 85.6 \text{ kpsi} \) \textit{Ans.}

The reduction in area is given by Eq. (2-12) is

\[ R = \frac{A \sigma A_i}{A_0} (100) = \frac{0.1987 - 0.1077}{0.1987} (100) = 45.8\% \] \textit{Ans.}

<table>
<thead>
<tr>
<th>Data Point</th>
<th>( P_i )</th>
<th>( \Delta l, A_i )</th>
<th>( \int )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>0.0004</td>
<td>0.0020</td>
<td>5032</td>
</tr>
<tr>
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<td>0.00030</td>
<td>10065</td>
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<td>4</td>
<td>3000</td>
<td>0.001</td>
<td>0.00050</td>
<td>15097</td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td>0.0013</td>
<td>0.00065</td>
<td>20130</td>
</tr>
<tr>
<td>6</td>
<td>7000</td>
<td>0.0023</td>
<td>0.00115</td>
<td>35227</td>
</tr>
<tr>
<td>7</td>
<td>8400</td>
<td>0.0028</td>
<td>0.00140</td>
<td>42272</td>
</tr>
<tr>
<td>8</td>
<td>8800</td>
<td>0.0036</td>
<td>0.00180</td>
<td>44285</td>
</tr>
<tr>
<td>9</td>
<td>9200</td>
<td>0.0089</td>
<td>0.00445</td>
<td>46298</td>
</tr>
<tr>
<td>10</td>
<td>8800</td>
<td>0.1984</td>
<td>0.00158</td>
<td>44285</td>
</tr>
<tr>
<td>11</td>
<td>9200</td>
<td>0.1978</td>
<td>0.00461</td>
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</tr>
<tr>
<td>12</td>
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<td>0.1963</td>
<td>0.01229</td>
<td>45795</td>
</tr>
<tr>
<td>13</td>
<td>13200</td>
<td>0.1924</td>
<td>0.03281</td>
<td>66428</td>
</tr>
<tr>
<td>14</td>
<td>15200</td>
<td>0.1875</td>
<td>0.05980</td>
<td>76492</td>
</tr>
<tr>
<td>15</td>
<td>17000</td>
<td>0.1563</td>
<td>0.27136</td>
<td>85551</td>
</tr>
<tr>
<td>16</td>
<td>16400</td>
<td>0.1307</td>
<td>0.52037</td>
<td>82531</td>
</tr>
<tr>
<td>17</td>
<td>14800</td>
<td>0.1077</td>
<td>0.84506</td>
<td>74479</td>
</tr>
</tbody>
</table>
\[ y = 3,05E+07x - 1,06E+01 \]

(a) Linear range

(b) Offset yield

(c) Complete range
(c) The material is ductile since there is a large amount of deformation beyond yield.

(d) The closest material to the values of $S_y$, $S_{ut}$, and $R$ is SAE 1045 HR with $S_y = 45$ kpsi, $S_{ut} = 82$ kpsi, and $R = 40\%$. \textit{Ans.}

2-7 To plot $\sigma_{true}$ vs. $\varepsilon$, the following equations are applied to the data.

$$\sigma_{true} = \frac{P}{A}$$

Eq. (2-4)

$$\varepsilon = \ln \frac{l}{l_0} \quad \text{for} \ 0 \leq \Delta l \leq 0.0028 \text{ in} \quad (0 \leq P \leq 8400 \text{ lbf})$$

$$\varepsilon = \ln \frac{A}{A_0} \quad \text{for} \ \Delta l > 0.0028 \text{ in} \quad (P > 8400 \text{ lbf})$$

where $A = \pi (0.503)^2 = 0.1987 \text{ in}^2$

The results are summarized in the table below and plotted on the next page. The last 5 points of data are used to plot log $\sigma$ vs log $\varepsilon$.

The curve fit gives $m = 0.2306$

$$\log \sigma_0 = 5.1852 \Rightarrow \sigma_0 = 153.2 \text{ kpsi} \quad \text{Ans.}$$

For 20% cold work, Eq. (2-14) and Eq. (2-17) give,

$$A = A_0 (1 - W) = 0.1987 (1 - 0.2) = 0.1590 \text{ in}^2$$

$$\varepsilon = \ln \frac{A}{A_0} = \ln \frac{0.1987}{0.1590} = 0.2231$$

Eq. (2-18): $S'_{y} = \sigma_{y} = 153.2(0.2231)^{0.2306} = 108.4 \text{ kpsi} \ \text{Ans.}$

Eq. (2-19), with $S'_{y} = 85.6$ from Prob. 2-6,

$$S'_{u} = \frac{S'_{u}}{1 - W} = \frac{85.6}{1 - 0.2} = 107 \text{ kpsi} \ \text{Ans.}$$
<table>
<thead>
<tr>
<th>$P$</th>
<th>$\Delta l$</th>
<th>$A$</th>
<th>$\varepsilon$</th>
<th>$\sigma_{\text{true}}$</th>
<th>$\log \varepsilon$</th>
<th>$\log \sigma_{\text{true}}$</th>
</tr>
</thead>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1000</td>
<td>0.0004</td>
<td>0.1987</td>
<td>0.0002</td>
<td>5032.71</td>
<td>-3.699</td>
<td>3.702</td>
</tr>
<tr>
<td>2000</td>
<td>0.0006</td>
<td>0.1987</td>
<td>0.0003</td>
<td>10065.4</td>
<td>-3.523</td>
<td>4.003</td>
</tr>
<tr>
<td>3000</td>
<td>0.0010</td>
<td>0.1987</td>
<td>0.0005</td>
<td>15098.1</td>
<td>-3.301</td>
<td>4.179</td>
</tr>
<tr>
<td>4000</td>
<td>0.0013</td>
<td>0.1987</td>
<td>0.00065</td>
<td>20130.9</td>
<td>-3.187</td>
<td>4.304</td>
</tr>
<tr>
<td>7000</td>
<td>0.0023</td>
<td>0.1987</td>
<td>0.00115</td>
<td>35229</td>
<td>-2.940</td>
<td>4.547</td>
</tr>
<tr>
<td>8400</td>
<td>0.0028</td>
<td>0.1987</td>
<td>0.0014</td>
<td>42274.8</td>
<td>-2.854</td>
<td>4.626</td>
</tr>
<tr>
<td>8800</td>
<td>0.1984</td>
<td>0.00151</td>
<td>44354.8</td>
<td>-2.821</td>
<td>4.647</td>
<td></td>
</tr>
<tr>
<td>9200</td>
<td>0.1978</td>
<td>0.00454</td>
<td>46511.6</td>
<td>-2.343</td>
<td>4.668</td>
<td></td>
</tr>
<tr>
<td>9100</td>
<td>0.1963</td>
<td>0.01215</td>
<td>46357.6</td>
<td>-1.915</td>
<td>4.666</td>
<td></td>
</tr>
<tr>
<td>13200</td>
<td>0.1924</td>
<td>0.03222</td>
<td>68607.1</td>
<td>-1.492</td>
<td>4.836</td>
<td></td>
</tr>
<tr>
<td>15200</td>
<td>0.1875</td>
<td>0.05802</td>
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<td>-1.236</td>
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<td></td>
</tr>
<tr>
<td>17000</td>
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<td>0.24002</td>
<td>108765</td>
<td>-0.620</td>
<td>5.036</td>
<td></td>
</tr>
<tr>
<td>16400</td>
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<td>0.41889</td>
<td>125478</td>
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<td>5.099</td>
<td></td>
</tr>
<tr>
<td>14800</td>
<td>0.1077</td>
<td>0.61245</td>
<td>137419</td>
<td>-0.213</td>
<td>5.138</td>
<td></td>
</tr>
</tbody>
</table>
2-8  Tangent modulus at $\sigma = 0$ is

$$E = \frac{\Delta \sigma}{\Delta \epsilon} \approx \frac{5000 - 0}{0.2 \times 10^{-3} - 0} = 25 \text{ (10^6 psi) } \text{ Ans.}$$

At $\sigma = 20$ kpsi
2-9  \( W = 0.20, \)

(a) Before cold working: Annealed AISI 1018 steel. Table A-22, \( S_y = 32 \text{ kpsi}, S_u = 49.5 \text{ kpsi}, \sigma_0 = 90.0 \text{ kpsi}, m = 0.25, f_r = 1.05 \)

After cold working: Eq. (2-16), \( f_r = m = 0.25 \)

Eq. (2-14), \( \frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25 \)

Eq. (2-17), \( \varepsilon = \ln \frac{A_0}{A_i} = \ln 1.25 = 0.223 < \varepsilon_u \)

Eq. (2-18), \( S_y = \sigma \varepsilon^m = 90 \left( 0.223 \right)^{0.25} = 61.8 \text{ kpsi} \quad \text{Ans. 93% increase} \quad \text{Ans.} \)

Eq. (2-19), \( \frac{S}{S_y} = \frac{49.5}{1-W} = 61.9 \text{ kpsi} \quad \text{Ans. 25% increase} \quad \text{Ans.} \)

(b) Before: \( \frac{S_u}{S_y} = \frac{49.5}{32} = 1.55 \quad \text{After:} \quad \frac{S_u'}{S_y} = \frac{61.9}{61.8} = 1.00 \quad \text{Ans.} \)

Lost most of its ductility.

2-10  \( W = 0.20, \)

(a) Before cold working: AISI 1212 HR steel. Table A-22, \( S_y = 28 \text{ kpsi}, S_u = 61.5 \text{ kpsi}, \sigma_0 = 110 \text{ kpsi}, m = 0.24, f_r = 0.85 \)

After cold working: Eq. (2-16), \( f_r = m = 0.24 \)
Eq. (2-14), \[
\frac{A_0}{A} = \frac{1}{1-W} = 1.25
\]
Eq. (2-17), \[
\varepsilon_i = \ln A_0 = \ln 1.25 = 0.223 < \varepsilon_f A_i
\]
Eq. (2-18), \[
S_y = \sigma \varepsilon^m = 110 (0.223)^{0.24} = 76.7 \text{kpsi} \quad \text{Ans. 174\% increase} \quad \text{Ans.}
\]
Eq. (2-19), \[
\varepsilon_i = \frac{S_y}{1-W} = \frac{61.5}{1-0.20} = 76.9 \text{kpsi} \quad \text{Ans. 25\% increase} \quad \text{Ans.}
\]
(b) Before: \[
\frac{S_y}{S_y} = \frac{61.5}{28} = 2.20 \quad \text{After:} \quad \frac{S_y'}{S_y} = \frac{76.9}{76.7} = 1.00 \quad \text{Ans.}
\]
Lost most of its ductility.

2-11 \(w = 0.20,\)
(a) Before cold working: 2024-T4 aluminum alloy. Table A-22, \(S_y = 43.0 \text{kpsi}, S_u = 64.8 \text{kpsi}, \sigma_0 = 100 \text{kpsi}, m = 0.15, f = 0.18\)
After cold working: Eq. (2-16), \(\dot{f} = m = 0.15\)
Eq. (2-14), \[
\frac{A_0}{A} = \frac{1}{1-W} = 1.25
\]
Eq. (2-17), \[
\varepsilon_i = \ln A_0 = \ln 1.25 = 0.223 > \varepsilon_f A_i \quad \text{Material fractures} \quad \text{Ans.}
\]

2-12 For \(H_B = 275,\) Eq. (2-21), \(S_u = 3.4(275) = 935 \text{MPa} \quad \text{Ans.}\)

2-13 Gray cast iron, \(H_B = 200.\)
Eq. (2-22), \(S_u = 0.23(200) - 12.5 = 33.5 \text{kpsi} \quad \text{Ans.}\)
From Table A-24, this is probably ASTM No. 30 Gray cast iron \quad \text{Ans.}\)

2-14 Eq. (2-21), \(0.5H_B = 100 \implies H_B = 200 \quad \text{Ans.}\)
For the data given, converting $H_B$ to $S_u$ using Eq. (2-21)

<table>
<thead>
<tr>
<th>$H_B$</th>
<th>$S_u$ (kpsi)</th>
<th>$S^2_u$ (kpsi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>230</td>
<td>115</td>
<td>13225</td>
</tr>
<tr>
<td>232</td>
<td>116</td>
<td>13456</td>
</tr>
<tr>
<td>232</td>
<td>116</td>
<td>13456</td>
</tr>
<tr>
<td>234</td>
<td>117</td>
<td>13689</td>
</tr>
<tr>
<td>235</td>
<td>117.5</td>
<td>13806.25</td>
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<td>13924</td>
</tr>
<tr>
<td>239</td>
<td>119.5</td>
<td>14280.25</td>
</tr>
<tr>
<td></td>
<td>$\Sigma S_u = 1172$</td>
<td>$\Sigma S^2_u = 137373$</td>
</tr>
</tbody>
</table>

Eq. (1-6)

$$\bar{S}_u = \frac{\sum S_u}{N} = \frac{1172}{10} = 117.2 \approx 117 \text{ kpsi} \quad \text{Ans.}$$

Eq. (1-7),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S^2_{u(i)} - N \bar{S}_u^2}{N-1}} = \sqrt{\frac{137373 - 10(117.2)^2}{9}} = 1.27 \text{ kpsi} \quad \text{Ans.}$$

For the data given, converting $H_B$ to $S_u$ using Eq. (2-22)

<table>
<thead>
<tr>
<th>$H_B$</th>
<th>$S_u$ (kpsi)</th>
<th>$S^2_u$ (kpsi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>230</td>
<td>40.4</td>
<td>1632.16</td>
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<tr>
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<td>40.86</td>
<td>1669.54</td>
</tr>
<tr>
<td>232</td>
<td>40.86</td>
<td>1669.54</td>
</tr>
<tr>
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<td>41.32</td>
<td>1707.342</td>
</tr>
<tr>
<td>235</td>
<td>41.55</td>
<td>1726.403</td>
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<td>1726.403</td>
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<td>1745.568</td>
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<td>42.47</td>
<td>1803.701</td>
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<tr>
<td></td>
<td>$\Sigma S_u = 414.12$</td>
<td>$\Sigma S^2_u = 17152.63$</td>
</tr>
</tbody>
</table>
Eq. (1-6)
\[ \bar{S}_u = \frac{\sum S_u}{N} = \frac{414.12}{10} = 41.4 \text{ kpsi } \text{Ans.} \]

Eq. (1-7),
\[ S = \sqrt{\frac{\sum_{i=1}^{N-1} S_u^2}{N-1}} = \sqrt{\frac{17152.63 - 10(41.4)^2}{9}} = 1.20 \text{ Ans.} \]

2-17 (a) Eq. (2-9)
\[ u_r \approx \frac{45.6^2}{2(30)} = 34.7 \text{ in \cdot lbf/\text{in} }^3 \text{ Ans.} \]

(b) \( A_0 = \pi(0.503^2)/4 = 0.19871 \text{ in}^2 \)

<table>
<thead>
<tr>
<th>( P )</th>
<th>( \Delta L )</th>
<th>( A )</th>
<th>( (A_0/A) - 1 )</th>
<th>( \int )</th>
<th>( \sigma = P/A_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0</td>
<td>0.000 4</td>
<td>0.000 2</td>
<td>5 032.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 000</td>
<td>0.000 6</td>
<td>0.000 3</td>
<td>10 070</td>
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</tr>
<tr>
<td>3 000</td>
<td>0.001 0</td>
<td>0.000 5</td>
<td>15 100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 000</td>
<td>0.001 3</td>
<td>0.000 65</td>
<td>20 130</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 000</td>
<td>0.002 3</td>
<td>0.001 15</td>
<td>35 230</td>
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<td></td>
</tr>
<tr>
<td>8 400</td>
<td>0.002 8</td>
<td>0.001 4</td>
<td>42 270</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 800</td>
<td>0.003 6</td>
<td>0.001 8</td>
<td>44 290</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 200</td>
<td>0.008 9</td>
<td>0.004 45</td>
<td>46 300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 100</td>
<td>0.196 3</td>
<td>0.012 28</td>
<td>45 800</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 200</td>
<td>0.192 4</td>
<td>0.032 80</td>
<td>66 430</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 200</td>
<td>0.187 5</td>
<td>0.059 79</td>
<td>76 500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17 000</td>
<td>0.156 3</td>
<td>0.271 34</td>
<td>85 550</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 400</td>
<td>0.130 7</td>
<td>0.520 35</td>
<td>82 530</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14 800</td>
<td>0.107 7</td>
<td>0.845 03</td>
<td>74 480</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the figures on the next page,
\[ u_r \approx \sum_{i=1}^{5} A_i \]
\[ = \frac{1}{2} (43 000)(0.001 5) + 45 000(0.004 45 - 0.001 5) + 81 000(0.4 - 0.059 8) + 80 000(0.845 - 0.4) \]
\[ \approx 66.7 \times 10^3 \text{ in \cdot lbf/\text{in} }^3 \text{ Ans.} \]
Shigley's MED, 10th edition

Chapter 2 Solutions, Page 14/22

2-18, 2-19 These problems are for student research. No standard solutions are provided.

2-20 Appropriate tables: Young's modulus and Density (Table A-5) 1020 HR and CD (Table A-20), 1040 and 4140 (Table A-21), Aluminum (Table A-24), Titanium (Table A-24c)

Appropriate equations:
For diameter, \[ \sigma = \frac{F}{A} = \frac{F}{(\pi/4)d^2} = S_y \quad \Rightarrow \quad d = \sqrt[3]{\frac{4F}{\pi S_y}} \]

Weight/length = \( \rho A \), Cost/length = $/in = ($/lbf)

Deflection/length = \( \frac{\delta}{L} = F/(AE) \)

With \( F = 100 \text{ kips} = 100(10^{3}) \text{ lbf} \),

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus</th>
<th>Density</th>
<th>Yield Strength</th>
<th>Cost/lbf</th>
<th>Diameter</th>
<th>Weight/length</th>
<th>Cost/length</th>
<th>Deflection/length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>units</td>
<td>Mpsi</td>
<td>lbf/in^3</td>
<td>kpsi</td>
<td>$/lbf</td>
<td>in</td>
<td>lbf/in</td>
<td>$/in</td>
</tr>
<tr>
<td>1020 HR</td>
<td>30</td>
<td>0.282</td>
<td>30</td>
<td>0.27</td>
<td>2.060</td>
<td>0.9400</td>
<td>0.25</td>
<td>1.000E-03</td>
</tr>
<tr>
<td>1020 CD</td>
<td>30</td>
<td>0.282</td>
<td>57</td>
<td>0.30</td>
<td>1.495</td>
<td>0.4947</td>
<td>0.15</td>
<td>1.900E-03</td>
</tr>
<tr>
<td>1040</td>
<td>30</td>
<td>0.282</td>
<td>80</td>
<td>0.35</td>
<td>1.262</td>
<td>0.3525</td>
<td>0.12</td>
<td>2.667E-03</td>
</tr>
<tr>
<td>4140</td>
<td>30</td>
<td>0.282</td>
<td>165</td>
<td>0.80</td>
<td>0.878</td>
<td>0.1709</td>
<td>0.14</td>
<td>5.500E-03</td>
</tr>
<tr>
<td>Al</td>
<td>10.4</td>
<td>0.098</td>
<td>50</td>
<td>1.10</td>
<td>1.596</td>
<td>0.1960</td>
<td>0.22</td>
<td>4.808E-03</td>
</tr>
<tr>
<td>Ti</td>
<td>16.5</td>
<td>0.16</td>
<td>120</td>
<td>7.00</td>
<td>1.030</td>
<td>0.1335</td>
<td>$0.93</td>
<td>7.273E-03</td>
</tr>
</tbody>
</table>

The selected materials with minimum values are shaded in the table above. Ans.

2-21 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three would favor steel, cast iron, or maybe a less common ferrous material. The expectation would likely be hot-rolled steel. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster.

From the measured weight of 7.95 lbf, the unit weight is determined to be

\[ w = \frac{W}{Al} = \frac{7.95 \text{ lbf}}{[\pi(1 \text{ in})^2/4](36 \text{ in})} = 0.281 \text{ lbf/in}^3 \]

which agrees well with the unit weight of 0.282 lbf/in^3 reported in Table A-5 for carbon steel. Nickel steel and stainless steel have similar unit weights, but surface finish and darker coloring do not favor their selection. To select a likely specification from Table A-20, perform a Brinell hardness test, then use Eq. (2-21) to estimate an ultimate strength.
of $S_v = 0.5H_b = 0.5(200) = 100$ kpsi. Assuming the material is hot-rolled due to the rough surface finish, appropriate choices from Table A-20 would be one of the higher carbon steels, such as hot-rolled AISI 1050, 1060, or 1080. Ans.

2-22 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous material like aluminum. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 2.90 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{2.9 \text{ lbf}}{\pi (1 \text{ in})^2 / 4}(36 \text{ in}) = 0.103 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.098 lbf/in reported in Table A-5 for aluminum. No other materials come close to this unit weight, so the material is likely aluminum. Ans.

2-23 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous copper-based material such as copper, brass, or bronze. To further distinguish the material, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 9 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{9.0 \text{ lbf}}{\pi (1 \text{ in})^2 / 4}(36 \text{ in}) = 0.318 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.322 lbf/in$^3$ reported in Table A-5 for copper. Brass is not far off (0.309 lbf/in$^3$), so the deflection test could be used to gain additional insight. From the measured deflection and utilizing the deflection equation for an end-loaded cantilever beam from Table A-9, Young’s modulus is determined to be

$$E = \frac{Fl^3}{3I_y} = \frac{100 (24)^3}{3(\pi (1)^{\frac{6}{4}}64)(17/32)} = 17.7 \text{ Mpsi}$$

which agrees better with the modulus for copper (17.2 Mpsi) than with brass (15.4 Mpsi). The conclusion is that the material is likely copper. Ans.

2-24 and 2-25 These problems are for student research. No standard solutions are provided.

2-26 For strength, $\sigma = F/A = S \Rightarrow A = F/S$
For mass, \( m = Al\rho = (F/S) l\rho \)

Thus, \( f_3(M) = \rho /S \), and maximize \( S/\rho \) \((\beta = 1)\)

In Fig. (2-19), draw lines parallel to \( S/\rho \)

\[\text{The higher strength aluminum alloys have the greatest potential, as determined by comparing each material’s bubble to the } S/\rho \text{ guidelines. Ans.}\]

2-27 For stiffness, \( k = AE/l \) \( \Rightarrow A = kl/E \)

For mass, \( m = Al\rho = (kl/E) l\rho = kl^2 \rho/E \)

Thus, \( f_3(M) = \rho /E \), and maximize \( E/\rho \) \((\beta = 1)\)

In Fig. (2-16), draw lines parallel to \( E/\rho \)
From the list of materials given, **tungsten carbide (WC)** is best, closely followed by aluminum alloys. They are close enough that other factors, like cost or availability, would likely dictate the best choice. Polycarbonate polymer is clearly not a good choice compared to the other candidate materials.  

**Ans.**

2-28 For strength,

\[
\sigma = \frac{Fl}{Z} = S \tag{1}
\]

where \(Fl\) is the bending moment and \(Z\) is the section modulus [see Eq. (3-26b), p. 104]. The section modulus is strictly a function of the dimensions of the cross section and has the units in \( \text{in}^3 \) (ips) or \( \text{m}^3 \) (SI). Thus, for a given cross section, \( Z = C(A)^{3/2} \), where \( C \) is a number. For example, for a circular cross section, \( C = \left( \frac{1}{\sqrt{\pi}} \right)^{-1} \). Then, for strength, Eq. (4 (1)) is

\[
\frac{Fl}{CA} = S \quad \Rightarrow \quad A = \left( \frac{Fl}{CS} \right)^{2/3} \tag{2}
\]
For mass, \[ m = A l \rho = \left( \frac{F l}{CS} \right)^{2/3} l \rho = \left( \frac{F}{C} \right)^{2/3} \frac{\rho}{S^{2/3}} \]

Thus, \( f_3(M) = \rho / S^{2/3} \), and maximize \( S^{2/3} / \rho (\beta = 2/3) \)

In Fig. (2-19), draw lines parallel to \( S^{2/3} / \rho \)

From the list of materials given, a higher strength **aluminum alloy** has the greatest potential, followed closely by high carbon heat-treated steel. Tungsten carbide is clearly not a good choice compared to the other candidate materials. *Ans.*

2-29 Eq. (2-26), p. 77, applies to a circular cross section. However, for any cross section *shape* it can be shown that \( I = CA \), where \( C \) is a constant. For example, consider a rectangular section of height \( h \) and width \( b \), where for a given scaled shape, \( h = cb \), where \( c \) is a constant. The moment of inertia is \( I = bh^3/12 \), and the area is \( A = bh \). Then \( I = h(bh^2)/12 = cb \ (bh^2)/12 = (c/12)(bh) = CA \), where \( C = c/12 \) (a constant).

Thus, Eq. (2-27) becomes
and Eq. (2-29) becomes

\[
m = Al \rho = \left( \frac{k}{3C} \right)^{1/2} l^{1/2} \left( \frac{\rho}{E} \right)^{2/3}
\]

Thus, minimize \( f_3(M) = \frac{\rho}{E^{1/2}} \), or maximize \( M = \frac{E^{1/2}}{\rho} \). From Fig. (2-16)

From the list of materials given, **aluminum alloys** are clearly the best followed by steels and tungsten carbide. Polycarbonate polymer is not a good choice compared to the other candidate materials. \( Ans. \)

| 2-30 | For stiffness, \( k = AE/l \) \( \Rightarrow \) \( A = kl/E \)  
For mass, \( m = Al \rho = (kl/E) l \rho = kl^2 \rho /E \)  
So, \( f_3(M) = \rho /E \), and maximize \( E/\rho \). Thus, \( \beta = 1 \). \( Ans. \) |
| 2-31 | For strength, \( \sigma = F/A = S \Rightarrow A = F/S \) |
For mass, \( m = Al\rho = (F/S)l\rho \)

So, \( f_3(M) = \rho /S \), and maximize \( S/\rho \). Thus, \( \beta = 1 \).  

\[ \text{Ans.} \]

2-32 Eq. (2-26), p. 77, applies to a circular cross section. However, for any cross section shape it can be shown that \( I = CA \), where \( C \) is a constant. For the circular cross section (see p. 77), \( C = (4\pi)^{-1} \). Another example, consider a rectangular section of height \( h \) and width \( b \), where for a given, scaled shape, \( h = cb \), where \( c \) is a constant. The moment of inertia is \( I = bh^3/12 \), and the area is \( A = bh \). Then \( I = h(bh^2)/12 = cb \cdot (bh^2)/12 = (c/12)(bh)^2 = CA \), where \( C = c/12 \), a constant.

Thus, Eq. (2-27) becomes

\[
A = \left( \frac{kl_3}{3CE} \right)
\]

and Eq. (2-29) becomes

\[
m = Al\rho = \left( \frac{k}{l^{5/2}} \right) \left( \frac{\rho}{E^{1/2}} \right) \left( \frac{A}{3C} \right)
\]

So, minimize \( f_3(M) = \frac{\rho}{E^{1/2}} \), or maximize \( M = \frac{E^{1/2}}{\rho} \). Thus, \( \beta = 1/2 \).  

\[ \text{Ans.} \]

2-33 For strength,

\[ \sigma = Fl/Z = S \quad (1) \]

where \( Fl \) is the bending moment and \( Z \) is the section modulus [see Eq. (3-26b), p. 104]. The section modulus is strictly a function of the dimensions of the cross section and has the units \( \text{in}^3 \) (ips) or \( \text{m}^3 \) (SI). The area of the cross section has the units \( \text{in}^{2/3} \) or \( \text{m}^{2/3} \). Thus, for a given cross section, \( Z = C(A)^{2/3} \), where \( C \) is a number. For example, for a circular cross section, \( Z = \pi d^{3/2} /32 \) and the area is \( A = \pi d^2 /4 \). This leads to \( C = (4\sqrt{\pi})^{-1} \). So, with \( \rho = Z = C(A)^{2/3} \), for strength, Eq. (1) is

\[
\frac{Fl}{3r^2} = S \quad \Rightarrow \quad \left( \frac{Fl}{CS} \right)^{2/3} = A
\]

\[ \text{Ans.} \]

For mass,

\[
m = Al\rho = \left( \frac{Fl}{CS} \right)^{2/3} l\rho = \left( \frac{F}{C} \right)^{2/3} l^{5/3} \left( \frac{\rho}{S^{2/3}} \right)
\]

So, \( f_3(M) = \rho /S^{2/3} \), and maximize \( S^{2/3} /\rho \). Thus, \( \beta = 2/3 \).  

\[ \text{Ans.} \]
2-34 For stiffness, \( k = AE/l \), or, \( A = kl/E \).

Thus, \( m = \rho Al = \rho (kl/E)l = kl^2 \rho /E \). Then, \( M = E/\rho \) and \( \beta = 1 \).

From Fig. 2-16, lines parallel to \( E/\rho \) for ductile materials include steel, titanium, molybdenum, aluminum alloys, and composites.

For strength, \( S = F/A \), or, \( A = F/S \).

Thus, \( m = \rho Al = \rho F/Sl = Fl \rho /S \). Then, \( M = S/\rho \) and \( \beta = 1 \).

From Fig. 2-19, lines parallel to \( S/\rho \) give for ductile materials, steel, aluminum alloys, nickel alloys, titanium, and composites.

Common to both stiffness and strength are steel, titanium, aluminum alloys, and composites. Ans.

2-35 See Prob. 1-13 solution for \( \bar{x} = 122.9 \) kcycles and \( s_x = 30.3 \) kcycles. Also, in that solution it is observed that the number of instances less than 115 kcycles predicted by the normal distribution is 27; whereas, the data indicates the number to be 31.

From Eq. (1-4), the probability density function (PDF), with \( \mu = \bar{x} \) and \( \sigma^2 = s_x \), is

\[
f(x) = \frac{1}{s_x \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - \bar{x}}{s_x}\right)^2\right] = \frac{1}{30.3 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 122.9}{30.3}\right)^2\right]
\]

(1)

The discrete PDF is given by \( f/Nw \), where \( N = 69 \) and \( w = 10 \) kcycles. From the Eq. (1) and the data of Prob. 1-13, the following plots are obtained.
<table>
<thead>
<tr>
<th>Range midpoint (kycles)</th>
<th>Frequency</th>
<th>Observed PDF</th>
<th>Normal PDF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$</td>
<td>$f$</td>
<td>$f/(Nw)$</td>
</tr>
<tr>
<td>60</td>
<td>2</td>
<td>0.002898551</td>
<td>0.001526493</td>
</tr>
<tr>
<td>70</td>
<td>1</td>
<td>0.001449275</td>
<td>0.002868043</td>
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<td>80</td>
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<td>0.004832507</td>
</tr>
<tr>
<td>90</td>
<td>5</td>
<td>0.007246377</td>
<td>0.007302224</td>
</tr>
<tr>
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<td>0.009895407</td>
</tr>
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<tr>
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<td>0.012809861</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
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<td>0</td>
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</tr>
<tr>
<td>210</td>
<td>1</td>
<td>0.001449275</td>
<td>0.00021141</td>
</tr>
</tbody>
</table>

Plots of the PDF’s are shown below.
It can be seen that the data is not perfectly normal and is skewed to the left indicating that the number of instances below 115 kcycles for the data (31) would be higher than the hypothetical normal distribution (27).